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COMMENT

Comment on the number of spiral self-avoiding walks

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Abstract. We show that the number of spiral self-avoiding walks on the square lattice is directly determined by the number of partitions of the integers. The next term in the asymptotic expansion is found, and numerous typographical errors in the original letter of Guttmann and Wormald are corrected.

In a recent paper (Guttmann and Wormald 1984, hereafter denoted GW) we determined the number of n-step spiral self-avoiding walks on the square lattice, s_n. The key equation given there was

$$s_n = \sum_{k=0}^{n-1} i_k c_{n-k} - \sum_{k=1}^{n-1} i_k i_{n-k},$$

with $i_0 = 1$, $c_0 = 0$, $c_1 = 1$ and $i_n = c_{n+1} - c_n$.

Recently Szekeres and Richmond made the surprising observation that c_n (the number of concatenatable spirals) is *precisely* the number of partitions of the integers. An elementary proof of this due to one of us (MH) is illustrated in figure 1. Figure 1(a) represents a partition of 13, and a line at 45° as shown splits the figure into two classes, representing vertical and horizontal segments. Thus we see that there are two horizontal segments of length 4 and 2 and two vertical segments of length 5 and 2.

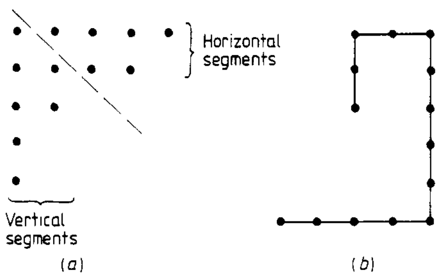

Figure 1.

The resultant spiral is shown in figure 1(b). Each partition can be uniquely represented by the pattern of points of the type shown in figure 1(a), and each such pattern corresponds to a distinct spiral walk. All spiral walks in class C are enumerated in this way.

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Szekeres has also calculated the correction term in equation (6) of GW which is

$$c = -\frac{1}{4\sqrt{3}}\left(\frac{\pi}{6\sqrt{2}} + \frac{6\sqrt{2}}{\pi}\right),$$

and from this result we have calculated the correction term to equation (9), which yields, for the coefficient of $1/\sqrt{n}$ in (9)

$$\beta = -\frac{35\sqrt{3}}{16\pi} - \frac{7\pi}{12\sqrt{3}} = -2.264\,08\ldots.$$

We also wish to point out a simple connection between 'live spirals' L_n and the class C_n, which is $c_n = l_{n+1} - l_n$.

Joyce (1984) has recently obtained the complete asymptotic expansion for s_n.

References

Joyce G S 1984 *J. Phys. A: Math. Gen.* **17** L463
Guttmann A J and Wormald N C 1984 *J. Phys. A: Math. Gen.* **17** L271

Corrigendum

On the number of spiral self-avoiding walks
Guttmann A J and Wormald N C 1984 *J. Phys. A: Math. Gen.* **17** L271–4

Line 3 of the abstract should read

$$s_n = \exp[2\pi(n/3)^{1/2}]c/n^{7/4}[1 + O(1/\sqrt{n})].$$

Equation (3) should read

$$s_n = \sum_{k=0}^{n-1} i_k c_{n-k} - \sum_{k=1}^{n-1} i_k i_{n-k}, \qquad \text{with } i_0 = 1,\ c_0 = 0,\ c_1 = 1.$$

Below equation (5), 'is defined in (1)' should be replaced by 'satisfies (1)'.

Equation (6) should read

$$c_n = (1/4\sqrt{3}\,.\,n)\exp[\pi(2n/3)^{1/2}][1 + c/\sqrt{n} + O(1/n)].$$

Equation (7) should read

$$i_n = (\pi/12\sqrt{2}\,.\,n^{3/2})\exp[\pi(2n/3)^{1/2}][1 + O(1/\sqrt{n})].$$